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Lateral Instability of a Stream of Charged Droplets

JOSEPH M. CROWLEY
Department of Electrical Engineering
University of Illinois, Urbana, Illinois
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Transverse displacements of a stream of charged droplets are unstable due to electrostatic repulsion. The instability is most pronounced at the shortest wavelength physically possible on the stream.

Streams of charged droplets find application in such diverse fields as space propulsion,[1] cloud physics,[2] and spray painting.[3] The success of all these applications depends on the possibility of controlling the droplet stream by external electric fields. If the droplets have a high charge to mass ratio, however, the stream exhibits an instability which causes it to diverge, as shown in Fig. 1 and which makes accurate control of individual droplets difficult.

An explanation for this divergence can be found in the electrostatic repulsion of the individual droplets. Consider an infinite stream of charged droplets of mass m, change q and spacing d. If any drop is moved slightly from the center line, it and its neighbor experience lateral forces (Fig. 2). This force can quickly be calculated under the assumption that only adjacent droplets interact. The lateral electric force on the nth droplet in Fig. 2 is approximately

$$f_e = (q^2/4\pi\epsilon_0 d^2)[(y_{n+1} - y_n)/d - (y_n - y_{n-1})/d],$$

if the droplets are replaced by point charges and the displacement y is much less than the spacing. The equation of motion of the nth droplet is then

$$m\frac{d^2y_n}{dt^2} = -\left(\frac{q^2}{4\pi\epsilon_0 d^3}\right)[y_{n+1} + y_{n-1} - 2y_n].$$

If a sinusoidal disturbance is assumed

$$y_n = Ae^{(\alpha t - jknd)};$$

a solution is possible only if k and α satisfy the relation

$$\alpha^2 = +(q^2/\pi\epsilon_0 md^3)\sin^2 kd/2,$$

in which the periodic nature of the droplet stream limits the allowed values of wavenumber to the range

$$0 \leq k \leq \pi/d.$$

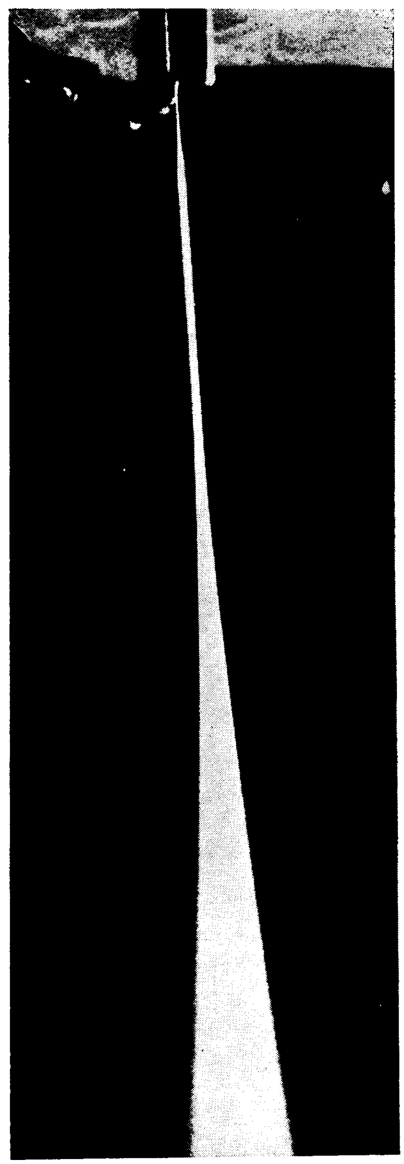

FIG. 1. The charged droplet stream exhibits a growing transverse disturbance (length shown is approximately 5 cm.)

This equation is similar to that for a massless elastic string, loaded at intervals with point masses[4] except for the minus sign in the right-hand term.

This dispersion equation gives real values for α, which indicates that the droplet stream is unstable for all allowed wavelengths. When the wavelength is very long, it includes many droplets, and the dispersion relation approaches the continuum limit, in which

$$\alpha^2 = (q^2/4\pi\epsilon_0 md)k^2.$$

Here the electrostatic forces furnish, in effect, a negative surface tension which drives the instability.

The growth rate is largest when k has its maximum value, and is given by

$$q^2/(\pi\epsilon_0 md^3).$$

Thus all allowed wavelengths are unstable, but there is a limit to the growth rate obtained.

This offers an explanation for the dispersion observed in Fig. 1, in which the jet was vibrated laterally in the process of generating the droplets. The instability appears in space because the droplet stream is moving downward at a velocity V_0. The spatial growth is closely given by

$$y \sim e^{\beta x},$$

when $\beta \simeq \alpha/V_0$.

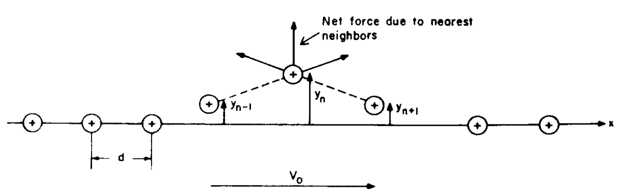

FIG. 2. The net electrostatic force on each droplet depends on the displacement of its nearest neighbors.

Experiments are now in progress to determine the mass, charge, velocity and spacing in the droplet stream, so that the growth rate can be checked accurately. The estimated parameters for the droplets

$$m = 10^{-10}\,\text{kg}, \qquad q = 10^{-14}\,\text{C},$$

$$V_0 = 10\,\text{m/sec}, \qquad d = 10^{-4}\,\text{m},$$

give a growth rate on the order of

$$\beta \approx 10^4/\text{m}.$$

This indicates an *e*-fold increase in the disturbance in 10^{-4} m, which is much faster than the increase indicated in Fig. 1. The theoretical expression, however, was derived under the assumption that the droplet displacement was much less than the spacing. As the disturbance grows, this spacing must increase, decreasing the electrostatic repulsion force which drives the instability, and thus decreasing the growth rate.

I would like to thank Simon Babil, who pointed out this instability and kindly furnished the photographs.

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