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LETTER TO THE EDITORS

On Mechanics of Bilayer Membranes

Evans and Simon, in their article "Mechanics of Bilayer Membranes," (1) make the statement: "... Young's modulus is only valid for a three-dimensional isotropic material." Whether this statement is true or not depends on the sense in which the name "Young's modulus" is used. In fact, Young's modulus is usually defined in conjunction with Hooke's law as the ratio of longitudinal stress to longitudinal extension in a simple tension or compression experiment. This definition was first given by Young (2) in 1807, repeated by Todhunter and Pearson (3) in 1886, by Love (4) in 1926, and appears in standard dictionaries, (5) encyclopedias (6), and textbooks (7).

In applying this definition, no assumption as the fluidity or isotropy of the material is required. Young himself gives the value of the modulus for air, (8) and Love (9) explicitly defines Young's modulus for an anisotropic material under simple tension as stress $= E$ (corresponding strain) when all other stress components vanish. For incremental extensions, the Young's modulus for simple tension in the z direction would be

$$\frac{1}{E_z} = \frac{\epsilon_{zz}}{\sigma_{zz}}\bigg|_{\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = \sigma_{xz} = \sigma_{xz} = 0.} \qquad [1]$$

Thus, Young's modulus has always been used to relate stress and strain under simple compression or tension, regardless of the properties of the material. Of course, much of the theoretical work in elasticity is concerned with isotropic solids, due to the relative simplicity of the elastic equations for this type of material. Under these assumptions, Young's modulus and the shear modulus may be related through the equation.

$$E = 9K_B\mu/(3K_B + \mu). \qquad [2]$$

For materials which are not isotropic solids, such as wood, air, and bilayers, this relation is not relevant, and the statement by Evans and Simon that the Young's modulus must be zero if the shear modulus vanishes implies an assumption of isotropic and solid behavior which need not be made.

Although this may appear to be a purely semantic difficulty, there is a real problem in the characterization of the elastic properties of a bilayer which Evans and Simon describe quite clearly. The elastic constants of an anisotropic material, however they are defined, never can be determined by a single measurement of stress and strain, unless the other stresses and/or strains are simultaneously measured or constrained. For example, in calculating the compression of a solvent bilayer, Evans and Simon make the assumptions that

$$\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = \sigma_{yz} = 0$$

to obtain the relation

$$-\sigma_{zz} = \sigma_0 = (4\lambda_a - 4\lambda_{az} + \lambda_z)\epsilon_{zz}. \qquad [3]$$

Recalling Love's definition (9) shows the relation between the elastic constants λ_a, λ_{az}, λ_z and Young's modulus for the z direction as

$$E_z = 4\lambda_a - 4\lambda_{az} + \lambda_z \qquad [4]$$

so that either Young's modulus or the elastic constants may be correctly used to describe this experiment. (Recently, Evans and Simon (10) have published a note on specific aspects of this type.)

Unfortunately, the measurements of bilayer compression reported in the literature do not, to my knowledge, include measurements of the lateral stresses imposed at the torus. Unless such measurements are included, these experiments cannot specify which of the many possible elastic constants is being measured. The best that can be done is to use the apparent relation between stress and strain as an estimate of the desired constant, as in my earlier paper (11).

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Received September 24, 1975; accepted November 19, 1975.