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EFFECT OF ELECTROMAGNETIC FORCE ON THE STABILITY OF LIQUID FILMS

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Steady electric and magnetic fields normal to each other and to the undisturbed surface are applied to a film of poorly conducting liquid flowing down an inclined solid plane under the influence of gravity. This combination of fields exerts a force normal to the undisturbed surface of the film which either stabilizes or destabilizes the flow, depending on the relative directions of the two fields. This force is independent of the orientation of the film, and thus can be effective in stabilizing the flow in a vertical or inverted film. The electromagnetic force must be on the order of the gravitational force normal to the undisturbed surface to be significant.

A THIN film of flowing liquid forms an integral part of many heat and mass transfer devices used in rocket nozzles, boilers, and condensers. The report that the onset of flow instability noticeably increases the heat and mass transfer of these devices (*3, 5, 7, 9, 13*) has led many investigators to examine the conditions which give rise to the various flow regimes (*2, 4, 11, 13, 14*). In the past few years, research in this field has been strongly stimulated by the development of theoretical analyses which explain most of the important features of the stability of thin films.

After Yih (*17*) and then Benjamin (*1*) attacked the basic problem successfully with a series solution, Yih (*15*) reproduced these results with a much simpler perturbation expansion in the wavenumber. The effects of changes in the physical properties of the flow were then investigated by Kao (*10*, two fluids), Whitaker (*12*, surface active agents), and Yih (*16*, non-Newtonian fluids). Most recently, Hsieh (*8*), in a study of the thin film analog of the Hartmann flow, showed that a magnetic field transverse to the flow direction can stabilize the film if the ratio of the magnetic force to the viscous force (Hartmann number) is very large.

In Hsieh's paper, the current induced by the motion of the liquid through the magnetic field interacts with the field to exert a decelerating force on the basic flow. This method suffers from two drawbacks. In many common liquids, the conductivity, and hence the induced current, is so small that the stabilizing force is negligible. In addition, since the force is proportional to the velocity of the fluid, this effect can only increase the critical Reynolds number by a factor which depends on the Hartmann number. If the flow is unstable for all Reynolds numbers, however, as in the vertical or inverted film, the flow cannot be stabilized.

In the present work, a force is exerted on the film perpendicular to the undisturbed free surface by applying electric and magnetic fields normal to each other and to the free surface. Since the current flow in the film is determined by the applied electric field, and not by the velocity, the film can be stabilized in any orientation.

Description of System

A thin film of liquid (Figure 1) flows down an inclined (with angle θ) solid plane under the influence of gravity. The liquid has a density ρ, a kinematic viscosity ν, an electrical conductivity σ, and a magnetic permittivity μ. In the steady state the film has a thickness d, and a velocity profile $V(y)$, which increases from zero at the solid to a maximum at the free surface.

A constant electric field transverse to the flow, E_a, drives an electric current, J, across the film. The insulated surface of the solid confines the current flow to the liquid. A constant magnetic field, B_a, in the flow direction interacts with the current to exert a body force normal to the solid on the film. The film surface suffers a small disturbance from this steady flow given by

$$y = d[1 + \xi(x,t)] \tag{1}$$

Equations of the System

The equations which describe the flow are the Navier-Stokes equation, modified to include the electromagnetic force

$$\frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \frac{\mathbf{J}\times\mathbf{B}}{\rho} + \mathbf{g} \tag{2}$$

and the equation of continuity for an incompressible fluid

$$\nabla \cdot \mathbf{u} = 0 \tag{3}$$

Of the equations which describe the electric and magnetic fields, only two are important here. One,

$$\mathbf{J} = \sigma\mathbf{E} + \sigma\mathbf{u}\times\mathbf{B} \tag{4}$$

describes the current flow in a moving medium. The first term represents the well known conduction current caused by an applied electric field. The second term states that the motion of a conductor through a magnetic field generates an electric current. This term plays a key role in the stabilization of a highly conducting film, as described by Hsieh (*8*). Here, however, there is an external current source, and this term only introduces complication into the analysis. Fortunately, it will usually be small, and can be neglected entirely under the condition

$$\frac{uB}{E} \ll 1 \tag{5}$$

For example, with the typical values

u = 10 cm. per sec.

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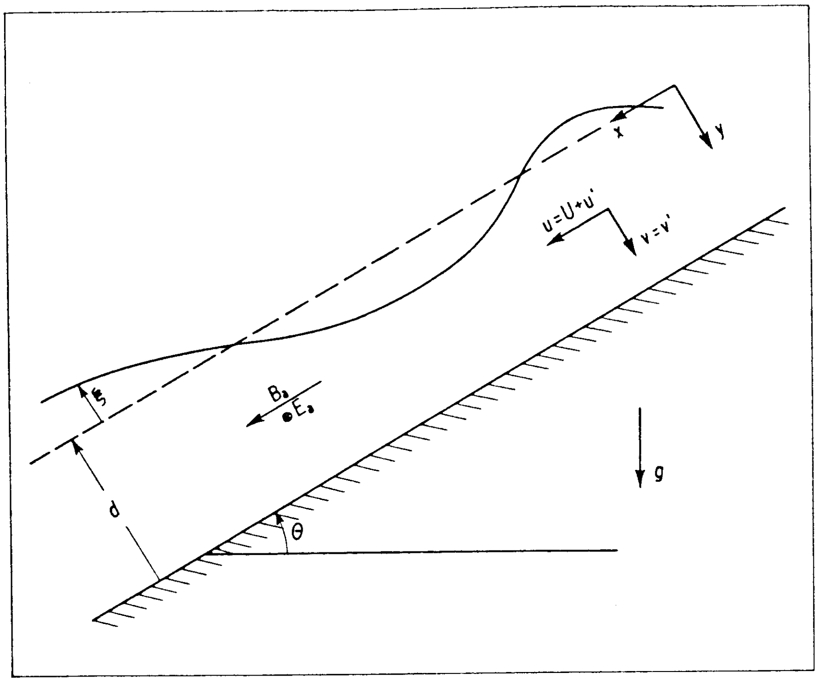

Figure 1. Thin film flowing down inclined plane under influence of gravity

Magnetic and electric fields are applied to exert a force normal to the plane

B = 1000 gauss = 0.1 weber per sq. meter
E = 100 volts per cm.

we find that

$$uB/E = 10^{-6}$$

The second electromagnetic equation, Ampere's law,

$$\int \mathbf{B}\cdot d\mathbf{r} = \mu \int\int \mathbf{J}\cdot d\mathbf{A} \qquad (6)$$

states that any current flow will set up a magnetic field. This induced field can be neglected if it is much less than the applied field, B_a, or if

$$\frac{\mu\sigma E_a d}{B_a} \ll 1 \qquad (7)$$

This inequality is easily satisfied because the film is so shallow. For example, if

μ = $4\pi \times 10^{-7}$ weber per ampere per meter
σ = 10 mhos per meter
d = 1 mm.
B_a = 1000 gauss

we find for Inequality 7

$$\mu\sigma\, E_a d/B_a \simeq 10^{-3}$$

These two inequalities can be combined to

$$R_m = \mu\sigma\, ud \ll \frac{uB_a}{E_a} \ll 1 \qquad (8)$$

where R_m is the magnetic Reynolds number.

With the neglect of the induced current and magnetic field, the electromagnetic force term in Equation 2 takes the form

$$(\mathbf{J} \times \mathbf{B})_y = \sigma E_a B_a \qquad (9)$$

This term, which is completely independent of the fluid velocity, represents a constant body force similar to gravity. The Navier-Stokes equations, written out in component form, are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u + g \sin\theta \qquad (10a)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v + g \cos\theta + \frac{\sigma E_a B_a}{\rho} \qquad (10b)$$

where the velocity vector has been split into its two components

$$\mathbf{u} = u\,\mathbf{i}_x + v\,\mathbf{i}_y \qquad (11)$$

The electric and magnetic fields in the bulk do not change the boundary conditions on the flow—namely, the vanishing of the normal and tangential velocity at the solid surface

$$u(y = d) = 0 \qquad (12a)$$

$$v(y = d) = 0 \qquad (12b)$$

and the continuity of the normal and shear stresses at the free surface

$$\frac{\partial u}{\partial y}(y = \xi) + \frac{\partial v}{\partial x}(y = \xi) = 0 \qquad (12c)$$

$$\frac{p}{\rho}(y = \xi) + 2\nu\frac{\partial v}{\partial y}(y = \xi) = 0 \qquad (12d)$$

Solution of Equations

To solve the equations we assume that the disturbance consists of two parts, a steady flow in the x-direction, and a small perturbation about this flow

$$u = U(y) + \tilde{u}(x,y,t) \qquad (13a)$$

$$v = \tilde{v}(x,y,t) \qquad (13b)$$

$$p = P(y) + \tilde{p}(x,y,t) \qquad (13c)$$

Substituting this relation into the equations and neglecting all terms containing products of the perturbation quantities, we obtain a set of linear equations, which are much easier to solve. In particular, the perturbation terms can be decomposed into fundamental solutions which take the form of waves traveling along the film in the x-direction. The film is then stable if each of these waves is stable.

In the basic flow the steady velocity and pressure are

$$U(y) = \frac{g \sin \theta}{2\nu}(d^2 - y^2) \tag{14}$$

$$P(y) = (\rho g \cos \theta + \sigma E_a B_a)y \tag{15}$$

We now simplify the problem by defining the dimensionless quantities

$$(x^*, y^*) = (x, y)/d \tag{16a}$$

$$t^* = t < u >/d \tag{16b}$$

$$(u^*, v^*) = (u, v)/<u> \tag{16c}$$

$$R = <u>\, d/\nu = \text{Reynolds number} \tag{16d}$$

$$p^* = p(\rho g d)^{-1} \tag{16e}$$

$$F_e = \sigma E_a B_a (\rho g)^{-1} = \text{electric Froude number} \tag{16f}$$

where

$$<u> = g d^2 \sin \theta/(3\nu) \qquad (17))$$

and the stream function

$$u^* = \frac{\partial \psi}{\partial y} \tag{18a}$$

$$v^* = -\frac{\partial \psi}{\partial x} \tag{18b}$$

The effect of an additional force normal to the plane, F_n, can be treated by adding it to the normal electromagnetic force, $\sigma E_a B_a$, in the electric Froude number. (The author is indebted to C. S. Yih for this suggestion.)

$$F_e = (\sigma E_a B_a + F_n)(\rho g)^{-1}$$

The remainder of the derivation is then unaltered.

We assume as the basic solution wavelike disturbances propagating along the film in the x-direction with a frequency ω, a wavenumber α, and a phase velocity $c = \omega/\alpha$

$$\psi(x^*, y^*, t^*) = \text{Re}\,[\varphi(y^*)\, e^{i\alpha(x^* - ct^*)}] \tag{19}$$

These assumptions reduce the problem to the solution of the Orr-Sommerfeld equation

$$\varphi'''' - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha R\,[(U - c)(\varphi'' - \alpha^2\varphi) - U''\varphi] \tag{20}$$

for the wave amplitude $\varphi(y^*)$. The boundary conditions are

$$\varphi'(1) = 0 \tag{21a}$$

$$\varphi(1) = 0 \tag{21b}$$

$$\varphi''(0) + (\alpha^2 - 3/c)\varphi(0) = 0 \tag{21c}$$

$$[\alpha\,(3 \cot \theta + 3F_e csc\, \theta)/\hat{c}]\varphi(0) + \alpha(R\hat{c} + 3\alpha i)\varphi'(0) - i\varphi'''(0) = 0 \tag{21d}$$

where $\hat{c} = c - 3/2$.

Since the instability first occurs at wavelengths much greater then the depth of the film, Equations 20 and 21 can be solved by a perturbation expansion in the wavenumber of the form

$$\varphi = \varphi_o + \alpha\varphi_1 + \ldots \tag{22a}$$

$$c = c_o + \alpha c_1 + \ldots \tag{22b}$$

This method, introduced by Yih (*15*), is sketched briefly in the next few paragraphs.

Substitution of the perturbation expansion into Equations 20 and 21, and collection of all terms in which α vanishes give the zero-order equation

$$\varphi_o'''' = 0 \tag{23}$$

with the boundary conditions

$$\varphi_o(1) = 0 \tag{24a}$$

$$\varphi_o'(1) = 0 \tag{24b}$$

$$\varphi_o''(0) - (3/\hat{c})\varphi_o(0) = 0 \tag{24c}$$

$$\varphi_o'''(0) = 0 \tag{24d}$$

Solution of Equations 23 and 24 gives the zero-order solution

$$\varphi_o = (1 - y)^2 \tag{25}$$

with the phase velocity

$$c_o = 3 \tag{26}$$

Collecting all terms containing α to the first power gives the first-order equation

$$\varphi_1'''' = iR[(U - 3)\varphi_o'' - U''\varphi_o] \tag{27}$$

with the boundary conditions

$$\varphi_1(1) = 0 \tag{28a}$$

$$\varphi_1'(1) = 0 \tag{28b}$$

$$\varphi_1''(0) - 2\varphi_1(0) + 4\hat{c}_1/3 = 0 \tag{28c}$$

$$2(\cot \theta + F_e\, csc\, \theta) - 3R - i\varphi_1'''(0) = 0 \tag{28d}$$

Solution of the first-order Equations 27 and 28 gives the phase velocity to the first order as

$$c = c_o + \alpha c_1 = 3 + i\alpha\left[\frac{6R}{5} - \cot \theta - F_e\, csc\, \theta\right] \tag{29}$$

The imaginary part of this dispersion relation, which indicates the stability of the basic flow, contains three terms. The first represents the viscous force which causes instability of the film. The second term, representing gravity, tends to stabilize the flow if $\theta < \pi/2$ and to destabilize it if the film is inverted ($\theta > \pi/2$). The last term represents the electromagnetic force.

Effect of Electromagnetic Force

The minus sign in the electromagnetic term does not mean that this force always tends to stabilize the flow. The sign results only from the choice of reference directions; reversing either the electric or magnetic field will destabilize the flow. This destabilization is impossible when the current is supplied by induction; in that case the electromagnetic force always tends to stabilize the flow.

The electromagnetic force, like that of gravity, is a body force independent of the velocity of flow. Unlike gravity, it is also independent of the inclination of the film. Thus the electromagnetic force can stabilize the flow when gravity is ineffective, as in the vertical film, or even when the gravity tends to destabilize the film, as in the inverted film. If the

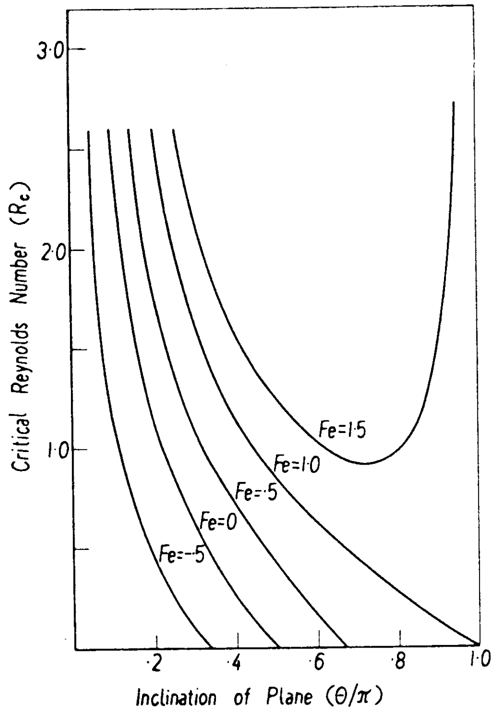

Figure 2. Stability of film

As the electromagnetic force, F_e, increases, the film is stable over a greater range of inclinations. As F_e decreases, the stable range also decreases

electric force is large enough ($|F_e| > 1$), it will always dominate the gravitational force for any inclination of the film. The direction of the fields can then be selected to make the film either unstable ($F_e < -1$) or stable ($F_e > 1$) for all orientations. Under these conditions, the gravity affects only the critical Reynolds number.

Another property of the electromagnetic force is that its magnitude can be changed by changing the applied voltage or magnetic field. This enables a convenient control of the flow stability and processes which depend on it, such as heat and mass transfer.

The effect of the electromagnetic force on the film stability as the orientation varies is shown by the graph of critical Reynolds number *vs.* angle of inclination for different electric Froude numbers (Figure 2). At $F_e = 0$, the electromagnetic force is absent. For negative electric Froude numbers, the maximum inclination for stability decreases, until at $F_e = -1$, the film is always unstable. For positive F_e the electromagnetic force tends to stabilize the film, until at $F_e = 1$, the film is stable for all orientations. When the film is either level ($\theta = 0$), or inverted ($\theta = \pi$), the liquid no longer flows, and the system reduces to that of Rayleigh and Taylor (*6*).

Conclusions

The stability of a poorly conducting, falling liquid film can be noticeably enhanced by the application of mutually perpendicular and magnetic fields if the electromagnetic force is on the order of the component of gravitational force normal to the undisturbed surface. Since this effect is independent of orientation, it can be used to stabilize the flow in a vertical or inverted film. The action can be reversed by reversing one of the fields, thus destabilizing the film and improving its heat and mass transfer characteristics.

Nomenclature

$\mathbf{A}$ = area vector, sq. meters
B = magnetic field, webers/sq. meter
c = ω/α, dimensionless wave speed
$\hat{c}$ = $c - 3/2$
D = complex quantity
d = depth of film, meters
E = electric field, volts/meter
F_e = $\sigma EB(\rho g)^{-1}$, electric Froude number
g = 9.80 meters/sec.^{2}, acceleration of gravity
$\mathbf{i}_{x,y}$ = unit vector in x,y direction
i = $\sqrt{-1}$
J = electric current density, amperes/sq. meter
p = pressure, kg./meter sec.$^{-2}$
P = pressure of basic flow, kg./meter sec.$^{-2}$
R = $\langle u\rangle d/\nu$, Reynolds number
R_c = critical Reynolds number
$\mathrm{Re}[D]$ = real part of complex quantity D
R_m = $\langle\mu\rangle d\sigma\mu$, magnetic Reynolds number
$\mathbf{r}$ = length vector, meters
t = time, seconds
U = velocity of basic flow, meters/sec.
u, v = x,y components of velocity, meters/sec.
$\mathbf{u}$ = velocity vector, meters/sec.
$\langle u\rangle$ = $gd^2 \sin\theta(3\nu)^{-1}$, average velocity of basic flow
x,y = spatial coordinates, meters

Greek Letters

α = dimensionless wave number
θ = angle of inclination, radians
μ = magnetic permittivity of liquid, webers/ampere/meter
ν = kinematic viscosity, sq. meters/sec.
ξ = dimensionless displacement of surface
π = 3.14159...
ρ = liquid density, kg./cu. meter
σ = electrical conductivity, mhos/meter
φ = y-dependent magnitude of stream function
ψ = stream function
ω = dimensionless angular frequency

Superscripts

$\sim$ = perturbation about basic flow
$*$ = dimensionless quantity

Subscripts

0 = zeroth term in wave number expansion
1 = first term in wave number expansion
a = denotes an externally applied field

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